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Strain paths during slaty cleavage formation—the role of volume loss

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Abstract—Strain analyses of slates commonly investigate the sequence of shapes that must have been superimposed to produce the observed strained state. Methods such as Rf/ϕ try to quantify the cleavage-forming deformation and 'see through' that component to examine any initial shape. An alternative approach is to reverse this process by considering the variation in final shapes derived when plausible cleavage-forming strain ellipsoids are superimposed on realistic initial shapes. Specifically, a deformation involving both plane strain and volume loss may be superimposed on an initial shape formed by diagenetic flattening. In this case the angle between bedding and the incipient cleavage plane is an important variable in controlling final ellipsoid shapes.

This approach has been adopted for a data set measured from the Ordovician Borrowdale Volcanics in the English Lake District. Results show that neither successive increments of plane strain without volume loss nor incremental plane strain plus volume loss adequately explain the observed pattern of final shapes. One permissible solution is generated if increments of pure volume loss perpendicular to the incipient cleavage plane precedes plane-strain distortion. Initial layer-parallel shortening may therefore play an important part during cleavage development in these slates.

INTRODUCTION

DEFORMED objects in rocks are useful as indicators of the total shape change the rock has suffered. These changes are due to many factors, including tectonic deformation and diagenetic compaction. The rigidity of an object compared with its matrix controls whether or not the object records the same deformation as its matrix. Accretionary lapilli in airfall tuffs have been successfully used as strain gauges because they appear to deform homogeneously with their matrix. In the British Caledonides accretionary-lapilli tuffs from the Ordovician of Snowdonia (Coward & Siddans 1979) and the Lake District (Helm & Siddans 1971, Oertel 1970, Bell 1981) have been used to give total deformation ellipsoids and in many cases these data have been extrapolated for the purposes of drawing regional conclusions.

Almost all studies of this type have aimed at factorizing the total shape recorded by the lapilli into component shapes, usually a component associated with cleavage formation and a pre-cleavage component. This is the basis of strain analysis techniques such as Rf/ϕ (Dunnet & Siddans 1971). The pre-cleavage component may be an initial variation in shape (e.g. Cloos 1947, Ramsay 1967, section 5.5) or it may be due to diagenetic compaction (Oertel 1970). In the case of accretionary lapilli from the Borrowdale Group of the Lake District it is probably both of the above (Bell 1981). The cleavage strain ellipsoid ($X_s > Y_s > Z_s$) is usually taken to be oriented in the cleavage frame (XY_s parallels the cleavage plane, X_s parallel to any stretching lineation), but there has been disagreement over whether the cleavage has been formed essentially by a plane strain ($K = 1$) (Oertel 1970) or by a flattening strain ($K < 1$) (Helm & Siddans 1971). A major drawback of any factorization method is that it can only resolve two component ellipsoids irrespective of the number of phases of strain the rock has undergone (Bell 1981).

There is an alternative approach to strain analysis in this situation. Instead of attempting to factorize ellipsoids from the total shape, it is possible to model the final shape ellipsoid by adding component ellipsoids together (Sanderson 1976). If a suite of ellipsoids measured from lapilli can be modelled by the superimposition of component ellipsoids that themselves represent realistic geological processes, then those processes were capable of forming the final lapilli shape. Conversely, if the superimposition does not produce an acceptable result we can conclude that those particular processes alone did not produce the final deformed state. This alternative approach relies on assumptions about the nature of the component ellipsoids that cannot be checked by examination of the final strained state, so at best the model is permissive not definitive.

In this paper four basic assumptions are made.

(1) The lapilli suffered diagenetic compaction. This point has been debated (Helm & Siddans 1971, Oertel 1970, Bell 1981), but the assumption that diagenesis produces an oblate spheroid oriented with its short axis normal to bedding seems geologically reasonable.

(2) The strains due to pre-cleavage folding can be ignored. This assumption is clearly unreasonable since folds which are demonstrably earlier than the cleavage occur (Soper & Numan 1974) and there must have been strains associated with their formation. This assumption produces an error which may be equivalent to 20% shortening (Bell 1981) and renders any model no more than an approximation numerically. Fold hinges are the least sensitive areas in this regard, so data from fold hinge areas must be given greater significance than those from fold limbs.

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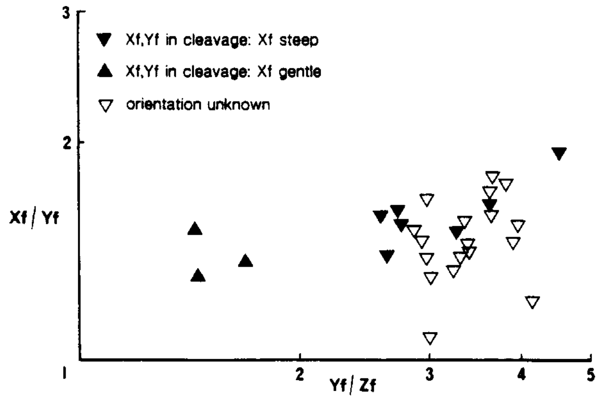

Fig. 1. Logarithmic Flinn plot showing average shapes of lapilli from all the measured localities in the Lake District. Data have been taken from Bell (1981). Inverted triangles represent final ellipsoids oriented close to the cleavage frame (XY_f near cleavage plane, X_f steeply plunging and close to the cleavage stretching lineation). Upright triangles represent ellipsoids with XY_f close to the cleavage plane but with gently plunging X_f axes, almost at right-angles to the stretching lineation.

(3) Any cleavage strain was oriented in the cleavage frame, that is XY_s parallel to the cleavage plane (Z_s normal to this plane) with X_s subvertical. Whether cleavage relates to the total finite strain or to some component strain has been extensively discussed (see, for example, Williams 1976 and Ghosh 1982) but here it is assumed that cleavage is related to deformation in this manner.

(4) The cleavage-forming strain was a combination of volume loss and plane strain. The validity or otherwise of this assumption is the subject of this paper.

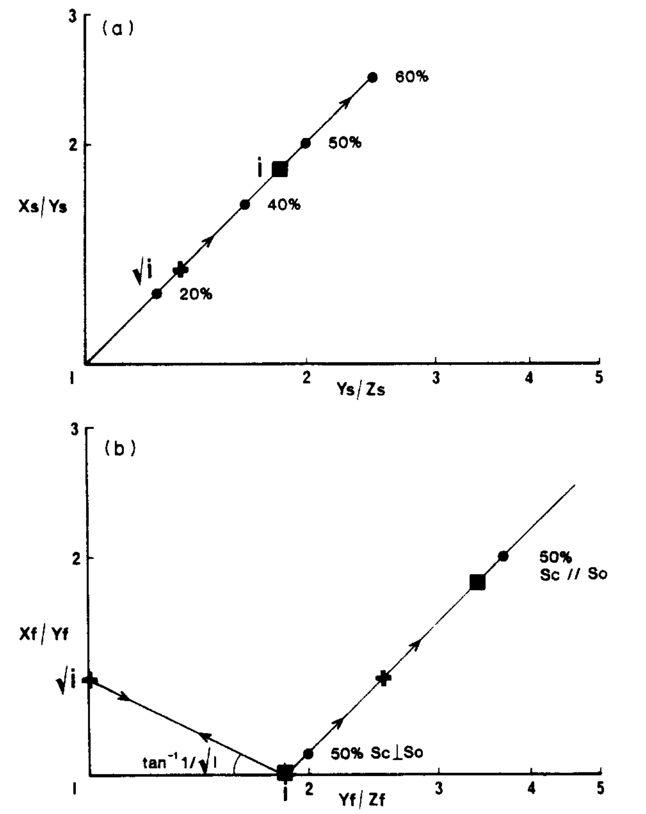

Fig. 2. Log Flinn plots showing tectonic strain path during cleavage formation (upper diagram) and shape path of final ellipsoid (lower diagram) when the strain path is superimposed on some initial shape, i. This model is for pure plane strain. Arrows indicate the progression of the total ellipsoid as successive increments are added. $X_s > Y_s > Z_s$ are the axes of the cleavage-producing strain ellipsoid. $X_f > Y_f > Z_f$ is the resultant total (final) ellipsoid after superimposition. Percentage shortening in Z is labelled, together with a shortening equivalent to the initial bedding compaction, i (solid square) and $\sqrt{i}$ (cross). Sc is the cleavage plane, So bedding.

BASIC DATA

The lapilli shapes that form the basis of this study are presented in Fig. 1. For full location and orientation details the reader is referred to Bell (1981), but note that the data were measured from both hinge zones and the common limb of a major pre-cleavage upright fold pair, and from localities separated by a strike length of over 25 km. X_f, Y_f and Z_f are the three principal axes of an averaged lapilli shape for each locality ($X_f > Y_f > Z_f$).

Two groups of data are apparent on Fig. 1. Most lapilli lie in the conventional field of slaty cleavage (Wood 1974, fig. 4) where the plane XY_f lies close to the cleavage plane and X_f is close to X_s, the cleavage stretching lineation. Since the observed stretching lineation is sub-vertical and pitches at a large angle on the cleavage plane to the sub-horizontal cleavage-bedding intersection, the lapilli X_f orientation may be described as 'X_f steep'. In contrast three data points, all from close to fold hinges, form a distinct grouping close to the origin of the plot and have X_f oriented near Y_s, close to the bedding-cleavage intersection. These have been called 'X_f gentle'.

STRAINS SUPERIMPOSED ON DIAGENETIC COMPACTION

Plane strain without volume loss

The effects of superimposition of a tectonic plane strain without volume loss on an initially oblate spheroid oriented in bedding have been examined by Ramsay & Wood (1973) and more exhaustively by Sanderson (1976). A tectonic plane strain path (Fig. 2a) superimposed incrementally on an oblate spheroid ($X/Y = 1$, $Y/Z = i$) gives a suite of possible final orientations, here called a final shape path (Fig. 2b).

Coaxial superimposition may be either constructive or destructive. If pre-tectonic bedding lies close to XY_s then the final shape path will have unit positive slope and intersect the Y_f/Z_f axis at i. The ellipsoids have been superimposed constructively and the average lapilli would be expected to record relatively large amounts of distortion (case 2 of Sanderson 1976). If bedding lies nearly at right angles to XY_s (case 1 of Sanderson 1976), destructive superimposition occurs and the final shape

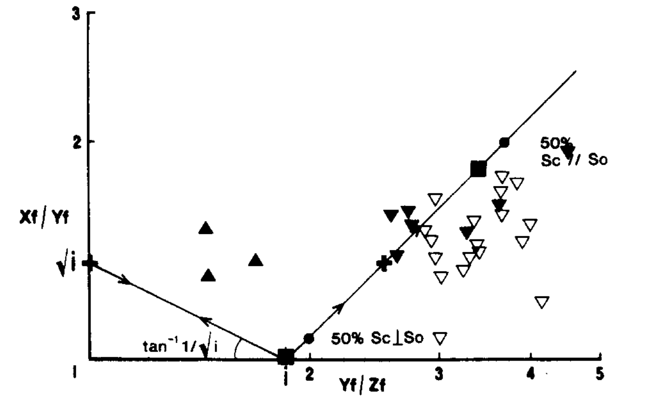

Fig. 3. Log Flinn plot showing a comparison of the predicted final shape path with the measured lapilli ellipsoids for a deformation by pure plane strain without volume loss. Data symbols as for Fig. 1. other symbols as for Fig. 2.

path moves firstly towards the prolate field until $Y_f = Z_f$ ($X_f/Y_f = \sqrt{i}$) and then back towards the abscissa, reaching the point $X_f/Y_f = 1$, $Y_f/Z_f = i$ when $X_s/Y_s = i = Y_s/Z_s$ (Fig. 2). Further tectonic deformation causes the final shape path to parallel the constructive superimposition path although for a given strain the destructive final shape lags behind the constructive final shape by a Y_f/Z_f value of i. The apparent 'bounces' or reflections on the X_f/Y_f or Y_f/Z_f axes indicate a switch in orientation of X_f or Z_f. The first bounce at $X_f/Y_f = \sqrt{i}$, $Y_f/Z_f = 1$ marks the relocation of Z_f from normal to bedding to normal to cleavage with X_f still sub-horizontal. At the second bounce point ($X_f/Y_f = 1$, $Y_f/Z_f = i$), X_f changes from sub-horizontal to sub-vertical. Bounce points represent a change in orientation of the final ellipsoid, not the tectonic ellipsoid. Sanderson noted that non-coaxial superimpositions produce a coincident final shape path—the apparent lag behind constructive superimposition varies from zero to $-i$ as the angle between bedding and incipient cleavage varies from 0 to 90°, respectively (Sanderson 1976 p. 43, Bell 1981, fig. 5).

Comparison of this predicted final shape path with the measured data is shown on Fig. 3. Whilst this model fits some data points in the 'X_f steep' field, the fit is poor for the three localities where the bedding-cleavage angle approaches 90°. These points should lie on the prolate to oblate section of the curve. Figure 3 shows the predicted final shape path for $i = 1.86$ (see Bell 1981 p. 473); for the path to include the three 'X_f gentle' localities i would need to be about 3.0. This amount of compaction (66%) seems unrealistically high for airfall tuff and in any case with this value as a starting point the coaxial constructive final shape path would miss all the rest of the data. This model is therefore unlikely to have been responsible for producing the data set of Fig. 1. Tectonic deformation must have been more complex than plane strain without volume loss.

Plane strain with incremental volume loss

Ramsay & Wood (1973) considered the effects of both initial volume loss (compaction) and incremental volume loss during tectonism. Tectonic deformation by plane strain with incremental volume loss is summarized both by Ramsay & Wood (1973 p. 271) and in Fig. 4. Incremental volume loss (Δ) is given by the expression

$$X_s Y_s Z_s = (1 - \Delta) \text{ (Ramsay \& Wood 1973, eqn. 9).}$$

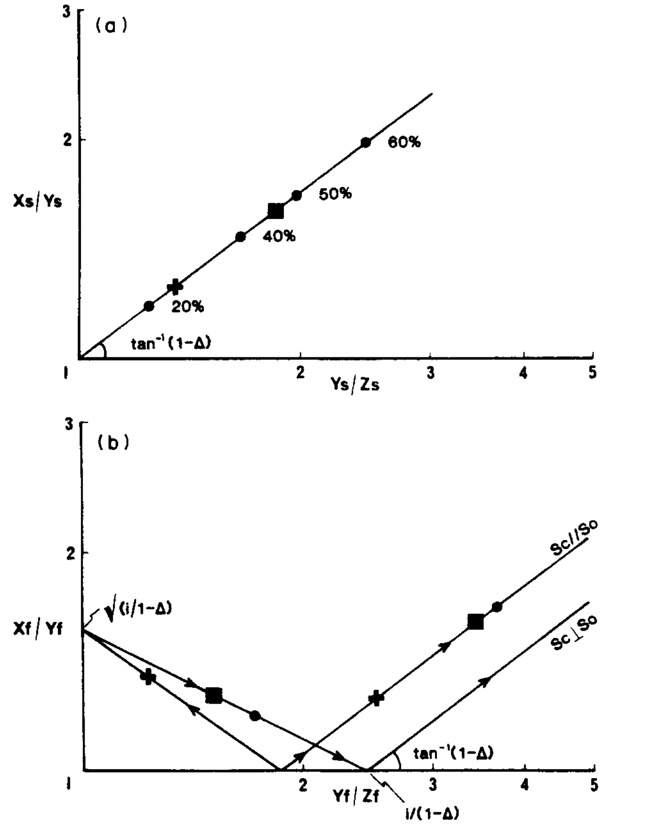

Fig. 4. Log Flinn plots of strain path (upper diagram) and final shape path (lower diagram) for a tectonic strain that is plane strain plus incremental volume loss. Symbols as in Fig. 2. Δ represents the volume loss, given by $XYZ = (1 - \Delta)$.

In Fig. 4 the strain path for plane strain with volume loss is similar to that in Fig. 2, except the slope of the path becomes $(1 - \Delta)$ instead of 1. When the deformation involves incremental volume loss the strain path is always in the field of apparent flattening. Figure 4 shows the final shape path for plane strain plus 20% incremental volume loss ($X_s.Y_s.Z_s = 0.8$). The form of the curve differs from that in Fig. 2 in four ways. Firstly, the constructive coaxial path makes an angle of 39° with the Y_f/Z_f axis, not 45°. Secondly, the destructive coaxial path bounces on X_f/Y_f at a greater X/Y value, $X/Y = \sqrt{i/1 - \Delta}$ ($=1.12i$ for $\Delta = 20\%$) compared with $X/Y = \sqrt{i}$ in Fig. 2. Thirdly, the second bounce of this path (X_f changes from gentle to steep) occurs at $Y_f/Z_f = i/1 - \Delta$ ($1.25i$ for $\Delta = 20\%$) compared with $Y/Z = i$ in Fig. 2. Fourthly, the remainder of the destructive coaxial path parallels the constructive coaxial path but is displaced towards the increased distortion part of the Flinn plot. Non-coaxial superimposition produces a family of curves of similar shape to the destructive coaxial curve, but bouncing at between Y_f/Z_f just greater than i for bedding-cleavage angles close to zero, and Y_f/Z_f approaching $i/1 - \Delta$ for bedding-cleavage angles approaching 90°.

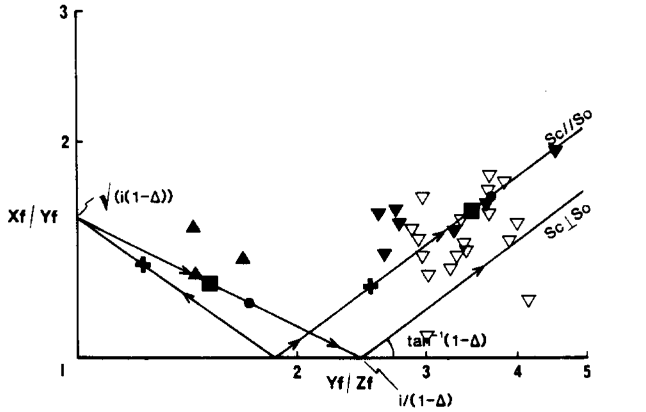

Fig. 5. Log Flinn plot showing comparison of predicted final shape path with measured data for plane strain plus incremental volume loss. Symbols defined in Figs. 1 and 2.

Comparison of the predicted final shape path with the measured data is shown on Fig. 5. Once again the model final shape path does not intersect the three critical 'X_f gentle' points. To do so would require either an initial compaction of Y/Z = 2.5 (60% compaction) or an incremental volume loss of much greater than 20%. In both these cases the constructive coaxial part of the curve would miss the 'X_f steep' data because of too large an intercept on the abscissa in the case of increased compaction, or too shallow a slope in the case of greater incremental volume loss, or both. Whilst this model of incremental volume loss during deformation more closely models the observed data, it still does not completely account for all the observed final shapes.

Cleavage formation by pure volume loss

The previous model encourages us to consider what the effects would be if cleavage formed by volume loss alone. The rock deformation can be likened to squeezing a sponge. There would be no increases in the long or intermediate dimensions of the tectonic ellipsoid, only an incremental reduction in the short axis. Such a strain path is shown in Fig. 6. Deformation is by reduction of Z_s alone, X_s and Y_s remain constant with a X_s/Y_s value of 1. The ratio Y_s/Z_s increases with increasing volume loss.

The nature of the final shape paths in this model are also shown in Fig. 6. Constructive coaxial superimposition, like the previous models, follows the tectonic strain path, the intercept on the Y_f/Z_f axis being displaced by a value i as before. Destructive coaxial superimposition produces an X/Y bounce point at $X_f/Y_f = i$. This is because one of the lapilli axes (Y_f) is shortening incrementally whilst the others (X_f, Z_f) remain at their initial length. At some increment of volume loss (46% if i = 1.86) the original short axis and the incrementally shortening axis become equal. Further volume loss affects only the new Z_f axis; the ratio X_f/Y_f remains constant at a value i and Y_f/Z_f increases with deformation to yield a final shape path parallel to the Y_f/Z_f axis. Non-coaxial superimposition produces a family of curves intermediate between the two coaxial curves, shown in Fig. 6.

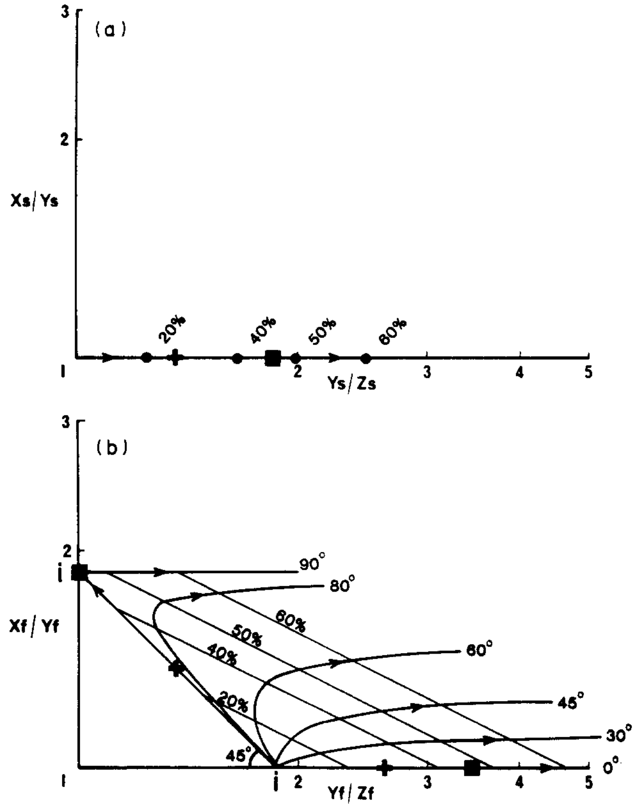

Fig. 6. Log Flinn plots showing strain path (upper diagram) and final shape paths (lower diagram) for a deformation that is pure volume loss normal to the incipient cleavage plane. Symbols defined in Fig. 2. Percentage values indicate volume loss in Zc, curves are labelled in degrees referring to the pre-strain bedding-cleavage angle.

Comparison of predicted final shapes with the measured data is shown in Fig. 7. This model is able to account for the three 'X_f gentle' localities since X_f remains parallel to the bedding-cleavage intersection all the time, but it cannot explain any of the 'X_f steep' localities since a steep X_f cannot be generated. Further, no explanation can be provided for the observed steep stretching lineation on the cleavage planes of these rocks. Another difficulty is that pure volume losses of greater than 60% seem unlikely as a cleavage-forming mechanism (Wood 1974 p. 391).

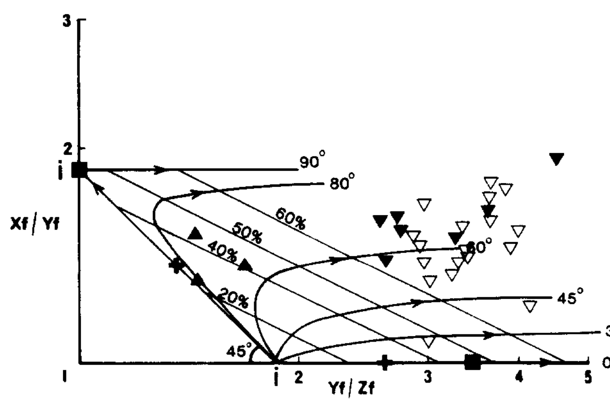

Fig. 7. Log Flinn plot showing the comparison of predicted final shape path with measured data for pure volume loss. Symbols as in Figs. 1 and 6.

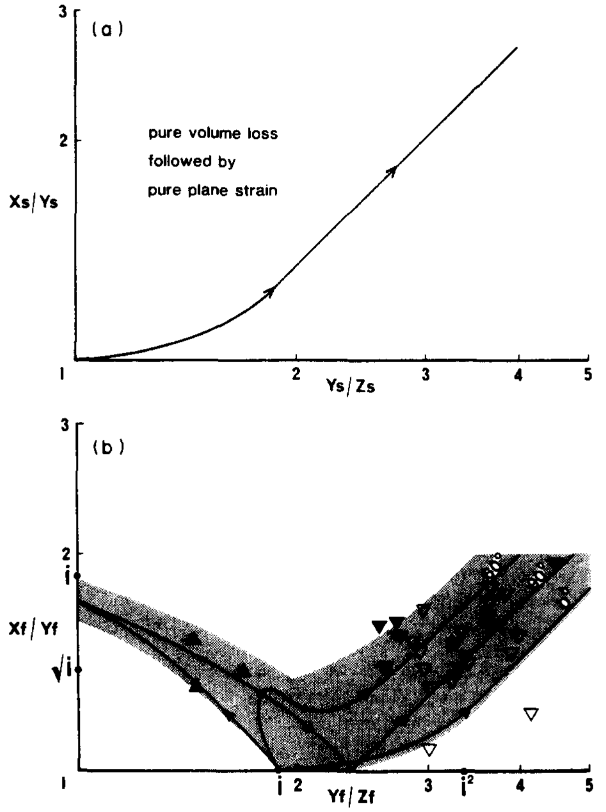

Fig. 8. Log Flinn plots showing the strain path (upper diagram) and final shape paths (lower diagram) for a cleavage-forming strain that is initially almost pure volume loss normal to the incipient cleavage plane, becoming almost pure plane strain in the advanced stages of deformation. Three final shape paths are shown for bedding-cleavage angles of 0, 60 and 90°. Shaded area indicates the zone of expected final shapes; the fit with the measured data both in magnitude and orientation is good.

Composite models

The accuracy with which the pure volume loss model accounts for one part of the data, together with the fact that the rest of the data fit the incremental volume loss model suggests that some composite tectonic deformation mechanism might be considered. Tectonic strains may have initiated as pure volume loss with no extension and shortening only in Z_s, to be replaced incrementally by plane strain with extension in X_s until at high strains deformation was by some plane strain mechanism alone. The tectonic strain path for such a model is shown in Fig. 8. The curve has an initial low positive gradient, steepening to a slope of 45° as plane strain totally replaces volume loss. The final shape paths produced when this strain is superimposed on an initial oblate ellipsoid are also shown in Fig. 8. The form of these curves is similar to those in the plane strain with incremental volume loss model except that the bounce point where Z_f becomes normal to cleavage is higher on the X/Y axis (between i and $\sqrt{i}$ depending on the amount of plane strain distortion) and the Y/Z bounce point where X_f becomes steep lies in the region Y_f/Z_f = i to i^2, depending on the amount of incremental volume loss. Note that these shape paths are not straight lines. The constructive coaxial path (labelled 0° in Fig. 8) defines a boundary to apparent flattening ellipsoids. The shaded area in Fig. 8 broadly represents the suite of final ellipsoids which would be expected from such a composite model; the data fit is adequate.

DISCUSSION

The arguments presented above imply that the suite of lapilli shapes from the Borrowdale Group tuffs is consistent with deformation by diagenetic compaction, followed by tectonism which initiated as a cleavage-normal volume reduction which gave way incrementally to plane-strain distortion. What independent checks can we perform to test this model further and what are the implications of the model for slate belt deformation?

Density is one independently measureable physical parameter which could be used to test the model. Wood (1974) has noted that many of the processes that are thought to contribute to the formation of slates are reflected by density changes. In particular, loss of pore space by repacking and rotation of phyllosilicates and dehydration during mineral phase changes, particularly low-temperature phyllosilicates altering to chlorite and white micas, can be expected to be major factors in volume reduction and density increase during slate formation. The primary process that would disturb the density-volume balance is mass transfer by pressure solution, but only when material has been removed from the system completely, and unfortunately these effects remain unquantifiable at present. Since the lapilli both lie in and are composed of airfall tuff, it is useful to compare density predictions from the above model with observed densities in welded and unwelded tuffs from recent volcanoes. Sparks & Wright (1979) have measured densities in tuffs from Quaternary volcanics in Greece. Welding in these airfall tuffs is caused by sintering of hot pumice and glass shards during compaction. Densities of approximately 0.8 g cm^{-3} are typical of the non-welded deposits. Welded units show higher densities, around 2 g cm^{-3} (Sparks & Wright 1979 p. 164). Volume loss during cleavage development must be responsible for an increase in density from 2 g cm^{-3} to 2.7 g cm^{-3}, which is the density of a typical slaty tuff from the Lake District. This is a volume loss of 26% and is in reasonable accord with the values shown in Fig. 7. The density changes presented in the composite model are at least plausible. It is interesting to note that this value is consistent with previous calculations (Wood 1974, fig. 5) but is a significantly higher volume loss than Wood's estimate for argillaceous slates.

This model shows that one tectonic strain path (Fig. 8) can account for the range of lapilli shapes seen in the localities sampled. Is it reasonable to assume that the strain pattern over 25 km of strike length could be this uniform? Comparison of Fig. 1 with the full location data (Bell 1981, fig. 3) indicates that the 'X_f steep' and 'X_f gentle' groupings are not spatially controlled. Two of the three X_f gentle localities are separated by a distance of 21 km along strike. Conversely, there is almost as much shape variation shown by the twenty samples that come from three X_f steep outcrops three kilometres apart as the whole of the X_f steep data set measured over many outcrops and over 20 km of strike. There is however a broad correlation between bedding-cleavage angle and the position on the shape plot (the nearer the origin, the greater the bedding-cleavage angle, (Bell 1981)) and between zones of steep bedding dip and intense cleavage development (Soper & Numan 1974). Since response of rocks to strain is a complex process it is surprising to find one tectonic strain path that can account for so many of the observed lapilli shape and cleavage-bedding features. In reality there will probably be as many strain paths as there are data points but the possibility exists that the curve in Fig. 8 represents some basic form; the implication is that the suite of real strain paths for these samples each approximate to this theoretical curve.

A further point of some interest concerns the difference between slates formed from mudrocks and slates formed from tuffs. In the Cambrian mudrocks of North Wales, Wood & Oertel (1980) have shown that the principal planes both of ellipsoidal spots and the preferred phyllosilicate orientations lie exactly parallel to the cleavage plane. Where data exist for the Borrowdale tuffs the cleavage plane is never exactly coincident with the principal plane of the average lapilli ellipsoid. The small difference between the two can consistently be attributed to a pre-strain factor which was an oblate spheroid oriented in bedding. If this bedding ellipsoid is not just a function of the pre-diagenetic shape of the lapilli (discussed in Bell 1981) then there is a fundamental difference between slaty mudrocks which have a cleavage fabric composed of compaction and tectonic strain and slaty tuffs in which the cleavage fabric is independent of pre-tectonic effects.

In summary, the suite of lapilli shapes that can be measured from tuffs in the Borrowdale Group are consistent with formation by diagenetic compaction normal to bedding followed by tectonism which produced cleavage. This initiated as uniaxial volume reduction normal to the incipient cleavage plane, presumably by pore-space reduction, rotation of stable phyllosilicate grains and early metamorphic dewatering reactions, and was replaced incrementally by plane strain deformation, which essentially conserved volume, as the rock reached some specific state of grain contact and metamorphic regrowth. Simpler deformation models do not account adequately for the observed data.

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